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Fixed price plus rationing: an experiment

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Abstract This paper theoretically and experimentally explores a fixed price mechanism in which, if aggregate demand exceeds supply, bidders are proportionally rationed. If demand is uncertain, in equilibrium bidders overstate their true demand in order to alleviate the effects of being rationed. This effect is the more intense the lower the price, and bids reach their upper limit for sufficiently low prices. In the experiment we observe a significant proportion of equilibrium play. However, subjects tend to overbid the equilibrium strategy when prices are high and underbid when prices are low. We explain the experimental evidence by a simple model in which the probability of a deviation is decreasing in the expected loss associated with it.

Keywords Fixed price mechanism · Rationing · Experimental economics

JEL Classification C90 · D45

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1 Introduction

Prices are not always set such that the market clears. Instead, we often observe non-price rationing of buyers, for different reasons. In initial public offerings, for example, the seller frequently sets a price at which she expects excess demand in order to be able to reward information revelation by large investors with some preferential treatment. In other situations, where demand is uncertain, the seller might simply not be able to set the market clearing price. In this case, two main classes of mechanisms have been proposed as solution to this problem: *auctions and fixed price mechanisms.* As for the latter, since supply is fixed (and price is chosen before actual demand reveals), the mechanism has to include a rationing device in case demand exceeds supply.

While axiomatic properties of different rationing schemes have been explored extensively in the literature, strategic behavior of buyers who expect to be rationed has up to date received little attention.[1] The few papers that explicitly analyze incentives in market games that may involve rationing of buyers find that these mechanisms are often desirable for the seller. In case a common value is sold, Bulow and Klemperer (2002) show that prices which require rationing can even be optimal. Gilbert and Klemperer (2000) come to the same conclusion for situations where customers have to make sunk investments to enter a market. In a private values setting, Bierbaum and Grimm (2006) analyze a fixed price mechanism where buyers are proportionally rationed in case of excess demand. They find that, if total demand is uncertain, bidders overstate their true demand to alleviate the effects of being rationed in high demand scenarios. This allows the seller to set the fixed price at a rather high level, which yields the surprising result that the fixed price mechanism outperforms alternative selling mechanisms (such as a uniform price auction)[2] with respect to a variety of criteria: revenue, variability of revenue in different demand scenarios, and minimum revenue that is raised if demand turns out to be low.[3]

The above findings contribute to explaining the frequent use of mechanisms that involve rationing of buyers. However, we know from an extensive experimental literature on market institutions that often human behavior differs substantially from theoretical predictions, which may affect the relative performance of different mechanisms. This motivated us to experimentally study bidding behavior in a fixed price mechanism with proportional rationing (FPM) quite similar to the one analyzed in Bierbaum and Grimm (2006).

Our experimental design is based on a model where neither the buyers, nor the seller, know total demand due to uncertainty about the number of (identical) buyers.

[1] See, for example, Herrero and Villar (2001), or Moulin (2000). Here rationing usually occurs because the allocating authority is not allowed to use prices in order to ration, e.g. in bankruptcy problems if claims are known but exceed the pie to be distributed.

[2] Since Bierbaum and Grimm consider large markets, in their framework a uniform price auction is incentive compatible and therefore a very attractive mechanism. It has often been proposed as an alternative to fixed price mechanisms or bookbuilding in order to conduct initial public offerings but never has been widely established.

[3] Also Chun (1989), Dagan et al. (1997), Moreno-Ternero (2002) and Herrero (2003) look at rationing from a noncooperative perspective. Herrero et al. (2004) provide an experimental study on the strategic behavior induced by rationing in the context of bankruptcy problems.

The seller, who is endowed with a given quantity of a divisible good, sets a fixed price, and then, buyers are asked to submit a quantity bid at this price. They are proportionally rationed in case the total quantity demanded exceeds supply, otherwise they receive what they asked for.[4] In the experiment, we were interested only in buyers' bidding behavior. Therefore, the seller's role was played by a computer, i.e. in each round a price was randomly chosen from the range where demand for the good was positive, which allowed us to extract complete bid functions. We also study an "incentive compatible mechanism" (ICM), which only differs from FPM in that buyers are never rationed. Given that the two mechanisms only differ with respect to the presence of the rationing device, we used ICM as a control treatment of the experimental results on FPM.

Let us give a quick overview of our main results.

First, we show that Bierbaum and Grimm's (2006) theoretical results on FPM are maintained in the context of small markets (i.e. for a finite number of buyers). At high prices bidding truthfully is optimal and rationing never occurs; at low prices bidders demand the highest possible quantity and they are rationed in any demand scenario; at intermediate prices bidders overstate their true demand, but only moderately, and rationing only takes place when demand is high.

In the experiment subjects play extremely well in ICM, where truthful bidding emerges as unanimous behavior since the very beginning. In FPM, behavior converges to equilibrium for very high and very low prices, where the equilibrium strategy is relatively easy to figure out. For intermediate prices, where equilibrium play is strategically more complex, some noise remains. As time proceeds, bidders even move away a bit from the risk neutral equilibrium prediction in the direction of overbidding. From a seller's point of view, this increases the attractiveness of FPM, since revealed demand is even higher than theoretically predicted exactly in the interval the optimal fixed price is chosen from.

Although the explanatory power of the theory seems impressive (especially if compared with that of standard auction theory models), we identify two significant deviations from equilibrium behavior: at intermediate prices, behavior evolves in the direction of overbidding; at low prices we observe—contrary to the Risk Neutral Nash Equilibrium (RNNE) prediction—price sensitivity of bids and underbidding. We show that both observations are consistent with the hypothesis of noisy directional learning (Anderson et al. 1999), where bidders adjust their actions in the direction of higher expected profits but do so subject to some exogenous noise (where the probability of an error is decreasing in the associated expected loss).

The remainder of the paper is organized as follows. In Sect. 2 we analyze the theoretical properties of FPM and ICM. The experimental design is described in Sect. 3. Section 4 contains the experimental results. It is divided into two parts. Descriptive statistics are presented first, followed by some panel data regressions that check the robustness of equilibrium predictions. Section 5 then investigates whether a quantal response equilibrium analysis may explain the systematic discrepancies between

[4] This is basically the model analyzed in Bierbaum and Grimm (2006). The only differences are that Bierbaum and Grimm analyze large markets (whereas in our experiment the number of potential buyers is small). Moreover, they allow for different types of buyers.

theory and evidence. Conclusions and directions for future research are contained in Sect. 6.[5]

2 Theoretical background and hypotheses

Consider a seller who has a fixed quantity (normalized to 1) of a perfectly divisible good and does not know the number of potential buyers interested in the good. By analogy with our experimental conditions, let us assume that n, the number of buyers, is either 2 or 4, where the probability that n is 2 (4) is λ $(1-\lambda)$. Throughout the paper, we shall refer to the case of $n=2$ $(n=4)$ as the "low" ("high") demand scenario. We assume that all potential buyers are identical. In particular, each buyer i has decreasing linear demand for the good,

$$x_i(p) = 1 - p. \tag{1}$$

In what follows, we provide a theoretical analysis of two mechanisms, the Fixed Price Mechanism (FPM) and an Incentive Compatible Mechanism (ICM).

2.1 FPM

We model FPM as a 3-stage, 4-player game with incomplete information. At stage 0 Nature moves, deciding market size n. Either two or four players participate in the market. In a market with two players, players are labeled "1" and "2".[6] If $n=4$, they are labeled "1" to "4". In what follows we look at the payoff of the representative player 1, who participates in the market, not knowing the number of his competitors.

At the remaining two stages, the seller and the buyers move in sequence. At stage 1, the seller announces a fixed price and an upper limit on individual bids $(p, \overline{d}) \in [0,1] \times \mathbb{R}_+$. At stage 2, each participating buyer i announces the quantity he demands at the posted price, $d_i \in [0, \overline{d}]$, which we will call buyer i's bid. If aggregate bids fall short of supply, each buyer obtains his bid, otherwise buyers are proportionally rationed. Each buyer has to pay the posted price for each unit he receives.

We formally describe proportional rationing as follows. Let $d \equiv \{d_i\}$ be the vector of bids and denote by $d_{-i} \equiv \{d_j\}_{j \neq i}$ the vector of bids by i's opponents. Then, the aggregate bid is given by $\sum_{i=1}^n d_i$, $n \in \{2,4\}$. Under proportional rationing, buyer 1 who bids d_1 receives a final quantity of $d_1 Q^n(d)$, where (recall that supply was normalized to one)

$$Q^n(d) = \min\left\{1, \frac{1}{\sum_{j=1}^n d_j}\right\}, \quad n \in \{2,4\}. \tag{2}$$

[5]The proofs of the theoretical results in Sect. 2 and the experimental instructions are provided in an Appendix that can be downloaded from the online version of this article.

[6]In the experiment, the other two players participated in a separate two-player market. Since we analyze the decision of the representative player 1, we ignore the existence of this parallel market.

We can now specify the players' expected payoffs. We use the index "0" for the seller and consider the representative bidder "1". Now, for a given pair $(p, \overline{d})$, let $\pi_i : [0, \overline{d}]^4 \to \mathbb{R}$ denote player i's expected payoff, given by

$$\pi_0(d) = \lambda Q^2(d) \sum_{j=1}^{2} d_j \cdot p + (1-\lambda) Q^4(d) \sum_{j=1}^{4} d_j \cdot p \tag{3}$$

and

$$\pi_1(d_1, d_{-1}) = \lambda \int_0^{d_1 Q^2(d_1, d_{-1})} (1 - x - p) dx + (1-\lambda) \int_0^{d_1 Q^4(d_1, d_{-1})} (1 - x - p) dx. \tag{4}$$

The extension to mixed strategies of the payoff structure (4) is straightforward, once we assume that players mix independently. If $\delta_i \in \Delta([0, \overline{d}]) \equiv \Delta_i$ ($\delta_{-i} \in \Delta([0, \overline{d}]^3) \equiv \Delta_{-i}$) denotes a generic mixed strategy for player i('s opponents), with $\delta_i(d_i)$ ($\delta_{-i}(d_{-i})$) denoting the probability of bidding d_i (d_{-i}) under δ_i (δ_{-i}), then $\pi_1(\delta_1, \delta_{-1})$ defines player 1's expected profit of a generic mixed strategy profile. In the following analysis, however, we restrict our attention to pure strategy profiles.

2.1.1 *Stage 2: the bidding stage*

We begin by characterizing optimal bidding behavior given the price p and upper limit on bids $\overline{d} \geq 1$.

Proposition 1 (Equilibria of Stage 2) *Let*

$$p_e = \frac{1}{4}\frac{9+7\lambda}{3+5\lambda} \quad \textit{and} \quad p_m = \frac{1}{4}\frac{9-\lambda}{3+\lambda}.$$

(i) $p \in [\frac{3}{4}, 1]$: *unique equilibrium* $d_i(p) = 1 - p$ *for all* i.
(ii) $p \in [0, p_e)$: *unique equilibrium* $d_i(p) = \overline{d}$ *for all* i.
(ii) $p \in [p_e, \frac{3}{4})$:

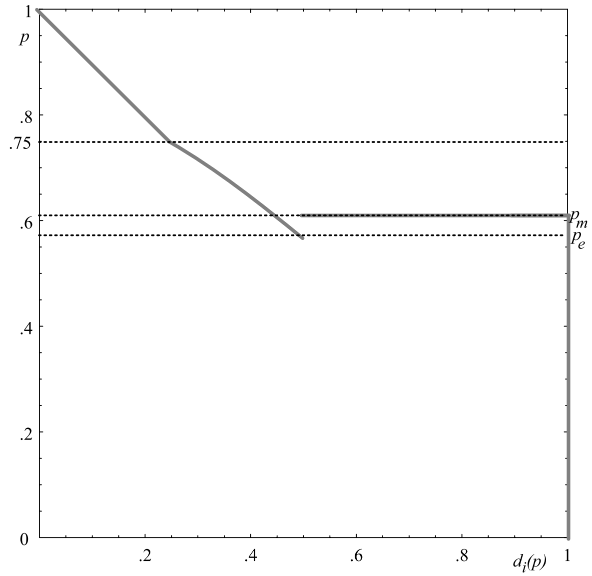
(a) $p \in (p_m, \frac{3}{4})$: *unique equilibrium*

$$d_i(p) = \frac{1}{2}(1-p) + \sqrt{\frac{1-\lambda}{\lambda}\left(\frac{3}{4} - p\right)\frac{3}{16} + \frac{1}{4}(1-p)^2} \quad \textit{for all } i.$$

(b) $p = p_m$: *a continuum of equilibria* $d_i = d$, $i = 1, \ldots, n$ *for all* $d \in [\frac{1}{2}, \overline{d}]$ *and one equilibrium where*

$$d_i(p) = \frac{1}{2}(1-p) + \sqrt{\frac{1-\lambda}{\lambda}\left(\frac{3}{4} - p\right)\frac{3}{16} + \frac{1}{4}(1-p)^2} \quad \textit{for all } i.$$

Fig. 1 Equilibrium bidding for $\lambda = \frac{1}{2}$



(c) $p \in [p_e, p_m)$: *two equilibria,* $d_i(p) = \overline{d}$ *for all* i *and*

$$d_i(p) = \frac{1}{2}(1-p) + \sqrt{\frac{1-\lambda}{\lambda}\left(\frac{3}{4} - p\right)\frac{3}{16} + \frac{1}{4}(1-p)^2} \quad \textit{for all } i.$$

Proof See the Appendix. □

Figure 1 suggests that the interval of possible prices can be split up into three subintervals:

- *High prices*: $p \in [\frac{3}{4}, 1]$. The buyers' aggregate demand never exceeds supply. Therefore, rationing plays no role and the buyers' optimal strategy simply is to bid truthfully.
- *Low prices*: $p \in [0, p_e)$. Large excess demand in the high demand scenario (and, at prices below $\frac{1}{2}$, also excess demand in the low demand scenario) yields a strong incentive to overstate true demand which leads to rationing in both scenarios. The only equilibrium is that every buyer asks for as much as possible, i.e. $\overline{d}$.
- *Intermediate prices*: $p \in [p_e, \frac{3}{4})$. Excess demand in the high demand scenario is moderate, which still yields an incentive to overstate demand. In the whole range of prices there is an equilibrium with moderate overbidding, where the optimal bids solve a trade-off between getting too much in the low demand scenario (where no rationing takes place) and getting too little in the high demand scenario (where buyers are rationed). When $p = p_m$, the game has also a continuum of symmetric (pure strategy) equilibria, one for every possible bid $d_i \in [\frac{1}{2}, 1]$. For prices $p \in [p_e, p_m)$ there exist two equilibria, the one with moderate overbidding and an equilibrium where demand explodes, like in the case of low prices.

2.1.2 Stage 1: Choice of the posted price and the upper-bound on bids

At stage 1 the seller chooses the profit maximizing price anticipating buyers' behavior at stage 2, but not knowing how many of them will participate in the market. We make two important observations:

(1) Only prices in the interval $p \in [p_e, \frac{3}{4}]$ can be rational choices of the seller. At p_e he sells the whole quantity in both demand scenarios in any equilibrium of stage 2 and it would definitely lower his profit if he posted a lower price. $p = \frac{3}{4}$ is the linear monopoly price given high demand and, since equilibrium bids are truthful at prices above $p = \frac{3}{4}$, a higher price cannot be profit maximizing under demand uncertainty.
(2) The seller will always choose the upper bound on bids $\overline{d}$ high enough not to affect revealed demand at the posted price.

Since the seller was simulated in our experiment, here we do without a detailed analysis of the seller's behavior. In an earlier version of this paper, Grimm et al. (2005), we formally show that any equilibrium of FPM has those two properties.

2.2 ICM

In our experiment we also tested another fixed-price mechanism, ICM, which only differs from FPM in that bidders always get what they ask for (i.e. there is no rationing). In ICM, player 1's payoff function (4) simplifies to

$$\pi_1(d_1, d_{-1}) = \int_0^{d_1} (1 - x - p)dx = \frac{1}{2} d_1(2 - 2p - d_1). \tag{5}$$

The absence of rationing breaks any strategic link among the players, who basically face a simple decision problem, whose solution is truthful bidding.

Proposition 2 *In ICM each bidder's optimal bid equals his true demand, i.e.*

$$d_i^*(p) = 1 - p. \tag{6}$$

In our experiment, ICM mainly serves as a robustness check for our experimental design, to evaluate whether subjects bid truthfully when it is a strictly dominant strategy to do so.

3 The experimental design

In what follows, we describe the experimental design in detail.

Subjects. The experiment was conducted in three subsequent sessions (two sessions devoted to FPM, and one to ICM) in May, 2004. A total of 72 students (24 per session) were recruited among the undergraduate student population of the Universidad de Alicante, mainly undergraduate students from the Economics Department with no (or very little) prior exposure to auction theory. The FPM sessions lasted approximately 120 min each, while the ICM session was slightly shorter (100 min approx.).

The experiment was computerized.[7] Subjects were given a written copy of the instructions in Spanish, together with a table indicating their monetary payoff associated with a grid of $21 \times 21 = 441$ representative price–quantity pairs.[8] Instructions were read aloud and we let subjects ask about any question they may have had. In addition, a self-paced, interactive computer program proposed three control questions, to make sure that subjects understood the main features of the game. In particular, we checked the comprehension of the rationing rule and the downward sloping demand function.

Treatment. In each session, subjects played 84 rounds of the corresponding mechanism. As for the FPM sessions, subjects were divided into three *matching groups* of 8. Subjects from different matching groups never interacted with each other throughout the session. As for the ICM session, every subject can be considered as a "matching group of size one".

Compared with the scale used in Sect. 2, in the experiment, all prices and quantities were multiplied by 10. We did this to mitigate "integer" frame problems.[9] Within each round $t = 1, \ldots, 84$, group size, composition and prices were randomly determined. Let *time interval* $T_k = \{t : 21(k-1) < t \leq 21k\}$, $k = 1, \ldots, 4$, be the subsequence of the kth 21 rounds. Within each time interval T_k, subjects experienced each and every possible price $p \in P = \{0, .5, 1, \ldots, 10\}$ (recall that the seller was simulated by the computer in our experiment). The sequence of those prices was randomly selected within each time interval and was different for each matching group. After being told the current price, subjects had to determine their bid, $d_i(p) \in [0, 10]$, for that round (subjects could not bid more than the entire supply).[10] With this design, we are able to characterize four complete individual bid schedules, one for each time interval. Moreover, in each round t, a (uniform) random draw fixed the group size $n \in \{2, 4\}$ independently for each matching group (i.e. $\lambda = \frac{1}{2}$). Given all these design features, we were able to collect 6 independent observations for FPM, and 24 for ICM.

Payoffs. All monetary payoffs in the experiment were expressed in Spanish ptas. (1 euro is approx. 166 ptas.).[11] Subjects participating in FPM (ICM) sessions received

[7]The experiment was programmed and conducted with the experimental software *z-Tree*, version 2.1 (Fischbacher 2007).

[8]The complete set of instructions, translated into English, can be found in the appendix at the journal's webpage.

[9]Nevertheless, in presenting our results, we shall not modify the scale to facilitate comparison with the content of Sect. 2.

[10]Bids were not constrained to be integer numbers. Instead, any possible quantity (up to three precision digits) was allowed.

[11]It is standard practice for all experiments run in Alicante to use Spanish ptas. as experimental currency. The reason for this design choice is twofold. First, it mitigates integer problems, compared with other currencies (USD or Euros, for example). On the other hand, although Spanish Pesetas are no longer in use (substituted by the Euro in the year 2002), Spanish people still use Pesetas to express monetary values in

2000 (1500) ptas. just to show up. These stakes were chosen to exclude the possibility of bankruptcy.

Ex-post information. After each round, subjects were informed of the payoff relevant information. For FPM this refers to group size, summary information on the aggregate behavior of their own group (both in terms of the total sum of individual bids, but also of the average bid(s) of the other member(s) of their group), the quantity of the good they actually received, together with the monetary profit associated with it. As for ICM, subjects were simply informed of the outcome of their individual bid (and the associated profit). All information was also given in the form of a *History Table*, so that subjects could easily review the results of all the rounds that they had played so far.

4 Results

In this section we report the results of our experiment. We begin by presenting some descriptive statistics which summarize the evolution of subjects' aggregate behavior over time in ICM and FPM. We then estimate dynamic panel data regressions. As for ICM, these regressions clearly show that equilibrium analysis almost perfectly explains subjects' behavior. Also for FPM our theoretical model provides a reasonably good prediction of actual behavior, even though our regressions unambiguously show persistent deviations from equilibrium. In short, in FPM people tend to overbid (underbid) the equilibrium strategy when rationing is less (more) severe.

4.1 Descriptive statistics

Figure 2 provides a graphical sketch of the evolution of the subjects' aggregate behavior for both experimental protocols, ICM (Fig. 2a) and FPM (Fig. 2b), tracing the average bids in the four experimental time intervals. The ordinate tracks prices, while the axis of abscissae reports average bids. The dotted line corresponds to the equilibrium strategy as given by Proposition 1; the four grey lines correspond to aggregate average bid functions per time interval, where greyscale is increasing with the time interval.

As Fig. 2a shows, subjects played ICM extremely "well". Their behavior is close to equilibrium from the very beginning, with some initial variance quickly vanishing over time. Out of 21 prices, in time interval 3 (4) *all* 24 subjects *always* played their dominant strategy in 19 (17) cases. Even though equilibrium play does not correspond to the subjects' unanimous decision, deviations from the dominant strategy are negligible and only observed for few subjects.

Things are different when we move to FPM, for which the evolution of aggregate bids is reported in Fig. 2b. Recall from Sect. 2 that in FPM the structure of equilibrium

their everyday life. In this respect, by using a "real" (as a opposed to an artificial) currency, we avoid the problem of framing the incentive structure of the experiment using a scale (e.g. "Experimental Currency") with no cognitive content.

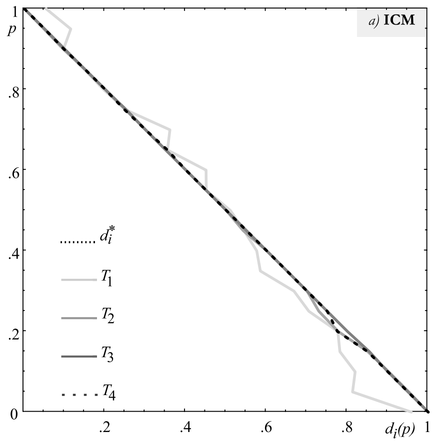


Fig. 2 Evolution of aggregate bids

bids crucially depends on the price level. Thus, we present our experimental evidence for three broad price intervals, which turn out to be crucial not only in the theoretical analysis, but also to evaluate subjects' behavior in the experiment:

- At high prices ($p \geq \frac{3}{4}$) truthful bidding corresponds to the unique equilibrium. We observe that subjects start bidding slightly more than their demand, and that overbidding gradually reduces over time.
- At low prices ($p < p_e$) demand explosion corresponds to the unique equilibrium. We find that individual bids get very close to the maximum possible amount of 1. However, contrary to the theoretical prediction, average bids seem to be sensitive to prices: the lower the price, the closer average bids get to the upper limit.
- At intermediate prices ($\frac{3}{4} > p \geq p_e \cong 0.568$) subjects start bidding above equilibrium and bids are increasing (i.e. moving away from equilibrium) over time.

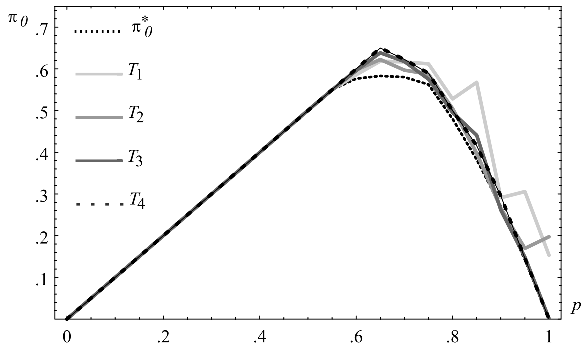
We finally look at the experimental evidence from the seller's viewpoint. Figure 3 plots the evolution of expected profits (ordinate) as a function of the ruling price given the observed behavior.[12]

As Fig. 3 shows, at low prices ($p \leq p_m$) actual profits equal their equilibrium levels. This is basically due to the fact that, within this price range, out-of-equilibrium underbidding is not sufficient to prevent subjects to be rationed in both demand scenarios. As a consequence, the entire supply is always sold, independently of the demand scenario. At high prices ($p \geq \frac{3}{4}$) expected profits start above equilibrium (due to overbidding), but converge quickly to their equilibrium level.

At intermediate prices ($p_e < p < \frac{3}{4}$) initial overbidding raises the seller's profits above their equilibrium levels. Moreover, since overbidding within this price range increases over time, also the seller's profit increases. Recall (see Sect. 2) that the profit-maximizing price always lies in the intermediate price range. Therefore, *persistent overbidding takes place exactly within the price range that would be selected*

[12]Note that, in the range $[p_e, p_m]$, FPM has multiple equilibria and, therefore, also seller's profit is not uniquely determined.

Fig. 3 FPM: evolution of the seller's profits



by a profit maximizing seller. In consequence, observed profit exceeds its equilibrium level in all four time intervals, and even increases (up to 12% above the theoretical prediction, since actual and predicted behavior lead to profits of .65 and .583 respectively).[13]

4.2 Panel-data regressions

In this section, our main concern is to check whether the discrepancies between observed and predicted behavior are statistically significant. To this aim, we construct a panel containing all decisions of all subjects in all rounds. Remember that each subject participated in 84 rounds of ICM (FPM), which creates a panel where subjects serve as the cross-sectional variable. The sample size is, therefore, 24 (48) subjects for ICM (FPM) session(s).

As for the ICM data, we use a simple random-effect linear regression. The underlying model assumes that subjects use linear bid functions, one for each time interval $T_k, k = 1, \ldots, 4$. Since individual (random) effects are common across time intervals, the model includes time interval dummies and their interactions with the ruling price p_t, as follows:

$$d_{it} = \sum_{k=1}^{4} \alpha_k \gamma_k + \beta_k \gamma_k p_t + \epsilon_i + \varepsilon_{it}, \tag{7}$$

where γ_k are time interval dummies (i.e. $\gamma_k = 1$ if $t \in T_k$); ϵ_i is the individual (random) component which describes subject i's unobserved time-invariant heterogeneity

[13] To illustrate the profitability of FPM, suppose that seller and buyers knew the market size, n. In such a case, the unique equilibrium would require the seller to set the linear monopoly price (i.e. either $p = \frac{1}{2}$ if $n = 2$, or $p = \frac{3}{4}$ if $n = 4$) and buyers to bid truthfully. Thus, the whole amount would be sold in both scenarios and the ex-ante expected revenue would be the expected monopoly profit $\pi_P = \frac{1}{2}\lambda + \frac{3}{4}(1 - \lambda)$. Since both scenarios are equally likely in our experiment (i.e. $\lambda = \frac{1}{2}$), $\pi_P = .625$. Since the theoretical expected revenue in FPM (.583) is lower than the expected linear monopoly profit theory predicts that the seller prefers a situation of full information. However, given the observed behavior the seller's profits are .65, which is higher than the expected monopoly profit.

Table 1 ICM: panel regression estimated coefficients

	Coef.	Std. Err.	z	$P > z$	[95% Conf. Int.]	
T1	0.853	0.054	15.680	0.000	0.740	0.965
T2	1.015	0.035	28.890	0.000	0.942	1.088
T3	1.034	0.033	31.550	0.000	0.966	1.102
T4	1.047	0.032	32.680	0.000	0.981	1.113
p*T1	−0.779	0.067	−11.550	0.000	−0.918	−0.639
p*T2	−0.995	0.037	−27.080	0.000	−1.071	−0.919
p*T3	−1.017	0.033	−31.180	0.000	−1.085	−0.950
p*T4	−1.033	0.032	−31.830	0.000	−1.100	−0.966

and ε_{it} is an idiosyncratic error term (we further assume $\epsilon_i \perp \varepsilon_{it}$). Table 1 reports the estimated coefficients of (7).

As Table 1 shows, bidders played closely to the assumed linear function in all time intervals. The overall R^2 for regression is .7878 (within-$R^2 = .8390$, between-$R^2 = 0$), which is an extremely high value when compared to panel regressions from similar experiments.

As for ICM, our null hypotheses correspond to $\alpha_k = 1$ and $\beta_k = -1$ for all k. In this respect, we first notice that only the estimates of α_1 and β_1 are significantly different from their theoretical predictions, both independently and jointly. This suggests that subjects started bidding less aggressively than predicted, with bids less sensitive to prices than expected. However, when we look at the estimated coefficients for $k > 1$, we find that we can never reject the null hypothesis that, from T_2 on, subjects played according to our theoretical conjecture. Neither we can reject the joint hypotheses $\alpha_2 = \alpha_3 = \alpha_4 = 1$ *and* $\beta_2 = \beta_3 = \beta_4 = -1$. This basically implies that learning mostly takes place in the first repetitions of the experiment and behavior stabilizes from T_2 on.

The FPM analysis is more complex and results are less straightforward. Table 2 reports estimates of a model which assumes that subjects are playing a piecewise linear bid function, as follows:

$$d_{it} = \sum_{k=1}^{4} \sum_{h=0}^{2} \gamma_k^h \alpha_k^h + \beta_k^h \gamma_k^h p_t + \epsilon_i + \varepsilon_{it}, \tag{8}$$

where γ_k^h is a dummy for time (as in (7)) *and* price interval, i.e. $\gamma_k^h = 1$ when $t \in T_k$ and $p_t \geq .75$ ($p_t \in (.55; .75)$) [$p_t \leq .55$], for $h = 0$ ($h = 1$) [$h = 2$], respectively. Observe that dummies γ_k^h partition the set of experimental conditions into the $4\times 3 = 12$ subsets that emerge from our theoretical analysis. In consequence, (8) estimates different—but, through the individual effects ϵ_i, interdependent—linear bid functions, one for each period and price subinterval. By the same token, β_k^h measures the sensitivity of bids on prices in all the subcases induced by the previous analysis.

Equation (8) can be interpreted as the natural extension of (7) to the case of FPM subject to some conditions, which we now discuss.

1. *Multiple equilibria.* Recall from Sect. 2.1 that there is a multiplicity of equilibria for $p \in [p_e, p_m]$. Given the price grid used in the experiment, multiplicity only occurs at $p = .6$, with equilibrium bids being 1 and .461, respectively.[14] In order to check which of these equilibria is somehow "more consistent" with our experimental evidence, we run two (independent) Wald tests with null hypotheses $H_1 : \overline{d}(.6) = .461$ and $H_2 : \overline{d}(.6) = 1$, respectively. While we cannot reject H_1, we can reject (at any meaningful confidence level) H_2. Consequently, we include $p = .6$ into the intermediate price interval.[15]
2. *Linearity.* The equilibrium bid function of FPM is not linear in the intermediate price interval (but concave). However, as Figure 1 shows, the demand function can be closely approximated by a linear function.
3. *Independent observations.* Unlike in ICM, where bidders always get what they ask for, in FPM they face competition of the other group members for their desired share. This, in turn, leaves open the possibility that the estimation of the variance-covariance matrix of (8)—not the estimation of the coefficients—needs to be adjusted to control for possible correlation among observations drawn from the same matching group. In other words, the estimations of Table 2 are performed under the assumptions that in FPM only the history of each matching group (and not the history of the 8 subjects that form each matching group) corresponds to an independent observation.[16]

Table 2 reports the estimation results. Again, subjects' behavior is close to the equilibrium bid function, although not as close as for ICM. This consideration notwithstanding, overall fit (.8048) is even higher (within-$R^2 = .8481$, between-$R^2 = .0164$). This is due to the higher number of regressors involved. Moreover, bidding behavior evolves quite differently for the different price intervals. Therefore, we discuss the results of the estimations separately for high, low, and intermediate prices.

- *High prices* ($p \geq \frac{3}{4}$). Here, the statistical analysis confirms the observation of Sect. 4.1 that behavior converges to truthful bidding. Estimated demand at $p = 1$, $\overline{d}(1)$, that is, $\alpha_k^0 + \beta_k^0$, $k = 1, \dots, 4$; drops from .044 in T_1 to $-.003$ in T_4. Price sensitivity (measured by β_k^0) slightly increases over time (i.e. overbidding decreases, as expected). From Table 2 we can detect significant differences between actual and predicted behavior only for time interval T_3. This is due to a drastic reduction in the standard error, rather than a significant shift in the estimated coefficients.
- *Low prices* ($p < p_e$). In this price interval the null hypotheses correspond to (i) $\alpha_k^2 = 1$ and (ii) $\beta_k^2 = 0$, $k = 1, \dots, 4$ (i.e. bids are independent of prices and coincide with the upper bound). As for (i), independent Wald tests do not reject the null in any case; as for (ii) the null hypothesis is always rejected, either independently or jointly. This basically implies that estimated demand at $p = 0$ (which moves from .94 to 1.035 from T_1 to T_4) is never lower than predicted. As for the

[14] Recall that prices were from the grid $\{0, .5, 1, \dots, 10\}$ and for $\lambda = \frac{1}{2}$ we get $p_e = .568$ and $p_m = .607$.

[15] P-values are .4787 for H_1 and 0 for H_2, respectively. In any case, we also run regressions excluding observation at $p = .6$. Results do not change (and are available on request).

[16] We thank an anonymous referee to point this out in an earlier version of this paper.

Table 2 FPM: panel regression estimated coefficients

	Coef.	Std. Err.	z	$P > z$	[95% Conf. Int.]	
T1*pHIGH	1.060	0.085	12.540	0.000	0.842	1.277
T2*pHIGH	0.950	0.161	5.890	0.002	0.535	1.365
T3*pHIGH	1.069	0.024	45.280	0.000	1.008	1.129
T4*pHIGH	1.114	0.046	24.360	0.000	0.996	1.231
T1*pINT	1.057	0.184	5.740	0.002	0.584	1.529
T2*pINT	1.364	0.164	8.340	0.000	0.944	1.785
T3*pINT	1.174	0.177	6.630	0.001	0.718	1.629
T4*pINT	1.603	0.195	8.240	0.000	1.103	2.103
T1*pLOW	0.940	0.025	37.600	0.000	0.876	1.004
T2*pLOW	1.007	0.014	70.870	0.000	0.971	1.044
T3*pLOW	1.003	0.028	35.410	0.000	0.930	1.076
T4*pLOW	1.035	0.023	44.590	0.000	0.976	1.095
p*T1*pHIGH	−1.016	0.099	−10.250	0.000	−1.271	−0.761
p*T2*pHIGH	−0.919	0.194	−4.750	0.005	−1.417	−0.421
p*T3*pHIGH	−1.070	0.025	−42.750	0.000	−1.134	−1.005
p*T4*pHIGH	−1.117	0.049	−23.030	0.000	−1.242	−0.992
p*T1*pINT	−0.954	0.262	−3.640	0.015	−1.628	−0.281
p*T2*pINT	−1.429	0.226	−6.310	0.001	−2.011	−0.847
p*T3*pINT	−1.110	0.269	−4.130	0.009	−1.801	−0.419
p*T4*pINT	−1.715	0.288	−5.940	0.002	−2.456	−0.973
p*T1*pLOW	−0.634	0.048	−13.180	0.000	−0.757	−0.510
p*T2*pLOW	−0.635	0.035	−18.300	0.000	−0.724	−0.546
p*T3*pLOW	−0.540	0.091	−5.940	0.002	−0.773	−0.306
p*T4*pLOW	−0.541	0.094	−5.750	0.002	−0.782	−0.299

evolution of the price sensitivity parameter β_k^2, Table 2 shows a sudden (downward) jump between the first and the last two time intervals (although the coefficients always remain statistically significant). Again, our observations of Sect. 4.1 are reinforced by our regression analysis: average bidding increases over time, although some (statistically significant) price sensitivity remains.

- *Intermediate prices* ($\frac{3}{4} > p \geq p_e \cong 0.568$). Within this price interval, the estimated bid functions coincide with equilibrium if (i) $\alpha_k^1 = 1.255$ and (ii) $\beta_k^1 = -1.324$, respectively. As for (i) the four estimated constants increase from 1.057 to 1.603, from T_1 to T_4 respectively; this basically implies that overbidding increases over time. As for (ii), Table 2 shows that also price sensitivity increases (from $-.954$ to -1.715 from T_1 to T_4 respectively). As a result of this joint movement, our evidence shows a significant shift in the estimated demand function in the direction of overbidding.[17]

[17]The joint test $(\alpha_1^1 = \alpha_4^1) \wedge (\beta_1^1 = \beta_4^1)$ is rejected at a 3% significance level.

To summarize, our panel-data analysis suggests that subjects behave almost perfectly in line with the equilibrium prediction in ICM and at high prices in FPM (where the equilibria of both games coincide with truthful bidding). For the remaining prices, the observed behavior in FPM differs from equilibrium. For intermediate prices, bids move away from equilibrium in the direction of overbidding. At low prices, bids are price sensitive, which yields underbidding relative to equilibrium. Although underbidding tends to disappear (the estimated inverse bid function shifts to the right), some price dependence remains.

5 Bounded rationality and out of equilibrium play

Our experimental results show that the equilibrium analysis developed in Sect. 2 is an (extraordinary) good predictor of subjects behavior (as far as ICM is concerned). This consideration notwithstanding, our regressions also show that subjects consistently deviate from equilibrium play, and that these deviations (in particular overbidding at intermediate prices and price sensitivity of bids at low prices in FPM) do not seem to vanish over time. To understand these empirical regularities of our experimental evidence, we hereafter assume that subjects are *boundedly rational*, i.e. that their choice is affected by some (unmodeled) external factors which make behavior intrinsically *noisy*. This noise may be induced by the complexity of the game, a limitation of the subjects' computational ability, random preference shocks, etc. This kind of choice framework may be modeled by specifying the payoff associated with a choice as the sum of two terms. One term is the expected utility of a choice, given the choice probabilities of other players. The second term is a random variable that reflects idiosyncratic aspects of payoffs that are not modeled formally.

Clearly, properties of this alternative class of models crucially depend on the specific way in which the stochastic process that generates noise is formally defined. One prominent recent approach is McKelvey and Palfrey's (1995) *quantal response equilibrium* (QRE). A quantal response is, basically, a "smoothed-out best response", in the sense that agents are not assumed to select the strategy that maximizes their expected payoff with probability one. Instead, each pure strategy is selected with some positive probability that increases in its expected payoff.[18]

Some recent papers (such as Anderson et al. 1999, or Goeree et al. 2002) have modified the notion of QRE to deal with games with a continuum of pure strategies, such as our ICM and FPM. A logit response function is often used to model the QRE. Formally, the standard derivation of the logit model is based on the assumption that payoffs are subject to unobserved preference shocks from a double-exponential distribution (e.g., Anderson et al. 1999). In this case, a (logit) QRE would be the fixed point

$$\delta_i(d_i) \equiv f_i(d_i|\delta_{-i},\mu) = \frac{\exp[\pi_i(d_i,\delta_{-i})\mu]}{\int_0^1 \exp[\pi_i(s,\delta_{-i})\mu]ds}, \quad i = 1,\ldots,4, \tag{9}$$

[18] See also Rosenthal (1989).

Table 3 Maximum likelihood estimations of μ

T_k	1	2	3	4
ICM	31.3	818.7	201200	1325.4
FPM	46	79	98	150

where $\pi_i(d_i, \delta_{-i})$ is the expected payoff associated with the pure strategy d_i against $\delta_{-i} \in \Delta_{-i}$, and μ is the noise parameter. As $\mu \to \infty$, the probability of choosing an action with the highest expected payoff goes to 1. Low values of μ correspond to more noise: if $\mu \to 0$, the density function in (9) becomes flat over the entire support and behavior becomes essentially random.

As we just argued, a (logit) QRE is a vector of densities that is a fixed point of (9). Continuity of the payoff function $\pi_i(\cdot)$ ensures existence, both in the case of ICM and FPM. While Sect. 5.1 explicitly characterizes the (unique) logit equilibrium in the case of ICM, for FPM no explicit solution can be found. This is because FPM is a game with a continuum of pure strategies, for which logit equilibria can be calculated only for very special cases.[19] In this case, we are only able to evaluate a QRE numerically. The equilibrium has the property that, when $\mu \to \infty$, it converges to the (unique) equilibrium we derived in Sect. 2. We use standard maximum-likelihood techniques to estimate the value of μ in each time interval and each treatment and report the results in Table 3.

5.1 ICM

Fix a price $p \in [0, 1]$ and consider the associated game induced by ICM. By (5), equilibrium distribution functions can be calculated as follows:

$$f(d_i|\mu) = \frac{\exp[\frac{\mu d_i(2-2p-d_i)}{2}]}{\int_0^1 \exp[\frac{\mu y(2-2p-y)}{2}]dy}. \tag{10}$$

The first row of Table 3 reports estimates of the noise parameter μ for ICM. The estimated noise parameter jumps dramatically between T_1 and T_2 and reaches its highest value at T_3 (precisely, 201200, basically infinity), implying that the empirical bid distribution collapses to the equilibrium one. The estimated value of μ then decreases in T_4, but is still significantly higher than in T_1. This confirms the finding that the learning mostly takes place in T_1.

Figure 4 shows, for each time interval, average bids observed in the experiment and bids in the estimated QRE (with the dotted line tracing the equilibrium strategy). The effect of noise (whose magnitude is measured by μ) is to create underbidding (with respect to equilibrium behavior) when the price is low(er than .5), and overbidding when the price is high(er than .5), independently of μ. The reason it that bidder profits are symmetric around $x_i(p)$, which implies that also the equilibrium distributions given by (10) are symmetric around the mode at $x_i(p)$. However, upwards

[19] Such as potential games, as in Anderson et al. (2001).

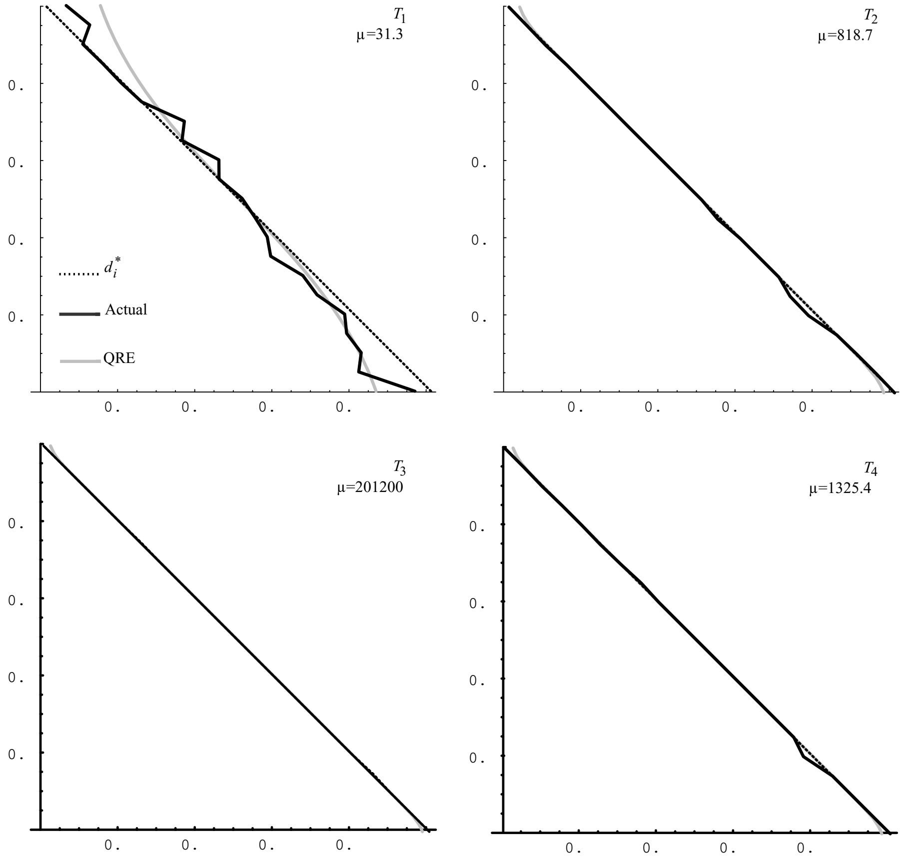


Fig. 4 ICM: evolution of the estimated QRE bid functions

(downwards) deviations are *more likely* for high (low) prices since the strategy space is restricted to the interval [0, 1].

As Fig. 4 shows, in T_1 our QRE analysis predicts well the slight overbidding (underbidding) when prices are high (low). The observed threshold where the average bid switches from overbidding to underbidding (as price decreases) is situated around $p = .5$, consistently with the QRE prediction. From T_2 on, the three curves almost coincide, as we know already from Sect. 4.

5.2 FPM

The last row of Table 3 reports the maximum-likelihood estimates of μ for FPM. A first look at Table 3 confirms the findings in Sect. 4: the estimated μ raises gradually from 46 in T_1 to a final 150 in T_4. This suggests that, as time proceeds, the observed behavior gets closer to the equilibrium prediction. Like in the case of ICM, the QRE analysis reproduces our experimental evidence with remarkable accuracy, as Fig. 5 shows.

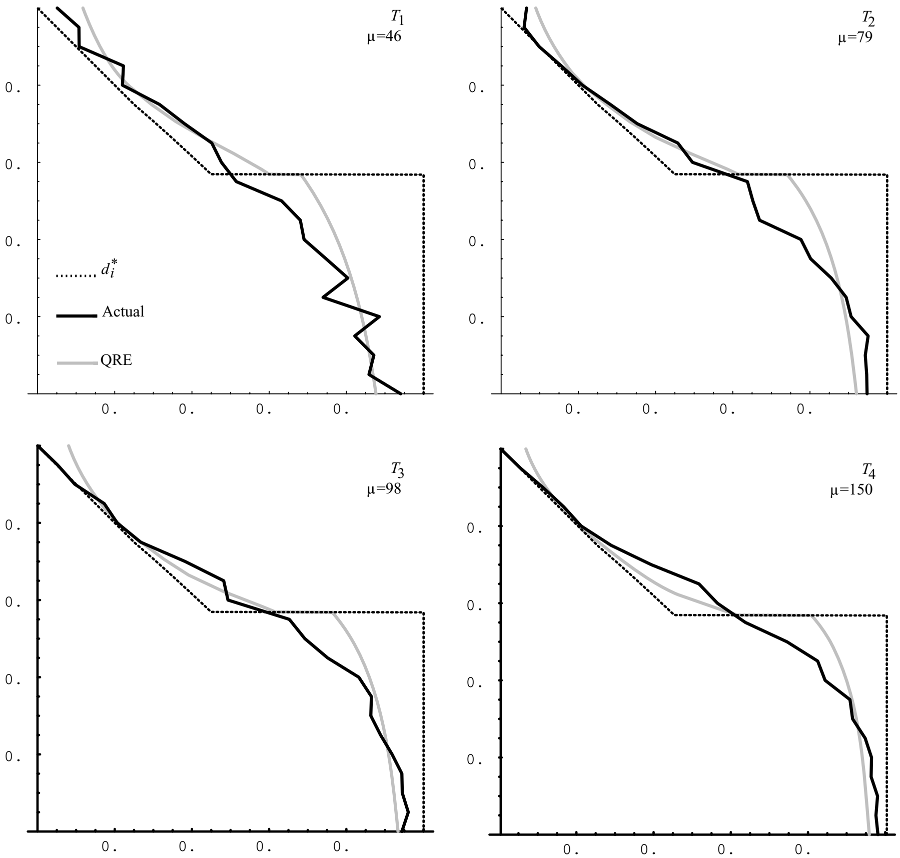


Fig. 5 FPM: evolution of the estimated QRE bid functions

Equilibrium distributions are analogous to the ICM case only for very high prices. In contrast, for very low prices, distributions are unimodal at 1. For prices $\frac{3}{4} > p \geq p_e$, the QRE distribution is not unimodal at $(1 - p)$, but has a mode at a higher level and is skewed to the right. That is, *deviations in the direction of overbidding are relatively cheaper* (and, therefore, by (9), overbidding with respect to d_i^* is more likely to occur). Furthermore, the larger the fraction of overbidding bidders is, the more attractive overbidding becomes for others. In other words, if overbidding strategies grow in probability, their payoff becomes relatively higher and this, by (9), reinforces the bias toward overbidding induced by the asymmetry in relative costs. In consequence:

- $\frac{3}{4} > p \geq p_e$. For intermediate prices, overbidding of the equilibrium strategy is more likely to be observed due to the deviation cost asymmetries highlighted in the previous paragraph.
- $p \geq \frac{3}{4}$. For very high prices, overbidding is basically due to the "drift effect" already discussed in Sect. 5.1.

- $p < p_e$. For very low prices, the drift effect yields underbidding. Moreover, QRE predicts the observed sensitivity of bids on price level (although sensitivity to prices is not as high as in our data). This is because the higher the price, the cheaper it is to underbid the equilibrium prediction by the same amount. Therefore, it is more likely to observe such deviations at higher prices.

6 Conclusion

Two main conclusions can be drawn from our experiment. First, compared to other experimental settings, equilibrium analysis provides a very good description of subjects' behavior. Second, there are still deviations from equilibrium, for which QRE (as opposed to risk aversion, for example) seems to produce a sufficiently consistent explanation.

A general and most important observation from our experimental data is that subjects were able to solve the problem well enough to achieve results closely resembling the theoretical predictions. This finding is important when it comes to the question when and where FPM should be used in practice. In this respect, two conclusions can be drawn. First, the theoretically appealing properties of FPM clearly survive (or even are improved on) in the laboratory, which suggests that FPM should be quite popular as a selling mechanism. Second, we have to keep in mind that those advantages of FPM can only be realized if the seller fixes the price correctly, anticipating buyers' bidding behavior. Thus, FPM should rather be observed in markets where sellers are experienced.

We emphasize that the observed deviations from equilibrium bidding make FPM even more attractive as a selling mechanism. Persistent overbidding of RNNE occurs exactly within the price range that would be selected by a profit maximizing seller. Revenue at this price turns out to be even higher than the expected monopoly profit of a seller who knows the demand scenario when he chooses the (linear) price.

Let us finally point to some interesting questions for future research. While in our experiment we focused on buyers' behavior, the seller's decision is certainly as relevant for evaluating the attractiveness of the mechanism. Two issues are of interest here. First, does the seller anticipate bidding behavior correctly and sets the price optimally given buyers' behavior? Second, does the fact that the seller is a real player (and not simulated by the computer) change buyers' behavior at the second stage of the game?

Another natural extension of the model studied in this paper could be the replacement of proportional rationing by a different rule. Two natural candidates are *constrained equal losses* and *constrained equal awards*. The former is a rule that makes the difference between what players ask and what players get as equal as possible across players, subject to the condition that no bidder ends up with a negative quantity. Under the constrained equal awards rule supply is distributed uniformly across bidders (i.e. they all receive the same amount), subject to the condition that no one gets more than her bid.

It is not difficult to show that the equilibria characterized in Sect. 2 maintain a similar feature if proportional rationing is replaced by constrained equal losses.[20] Only the intermediate price interval equilibrium may be slightly modified by the different rationing scheme. On the other hand, constrained equal awards affects the equilibrium considerably. In this case, a symmetric equilibrium is to submit the true demand for all $p > \frac{1}{2}$. For prices below this threshold, any (asymmetric) bid such that $\min\{d\} \geq \frac{1}{2}$ is an equilibrium. In other words, multiple equilibria occur for prices sufficiently low. How the presence of such strong strategic uncertainty may affect subjects' behavior in the lab is left for future research.

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[20] FPM with constrained equal losses affects the equilibrium properties if bidders are asymmetric, since different types have very different incentives to overstate demand.

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